Angular Dispersion

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The angle of diffraction depends on wavelength:

Gratings are special because they introduce dispersion to the diffracted light waves. A wave experiences dispersion when one of its features, such as velocity or direction, depends on its frequency or, equivalently, its wavelength. Perhaps the most widely used dispersive property of gratings is angular dispersion—the fundamental enabler for most spectroscopy measurements and instruments.

One need look no further than the Grating Equation [1] to understand angular dispersion:

$$\sin\theta_m = \sin\theta + m\lambda f \times 10^{-6} \quad (\lambda \text{ in nm}; f \text{ in lines/mm}). \tag{1}$$

For a given grating frequency f and angle of incidence θ, the Grating Equation shows how the diffracted angle θ_m for order m depends on the wavelength of light λ. Differentiating both sides of (1) with respect to λ, we find

$$\frac{d\theta_m}{d\lambda} = \frac{mf}{\cos\theta_m}, \tag{2}$$

or, in terms of the known quantities and wavelength,

$$\frac{d\theta_m}{d\lambda} = \frac{mf}{\sqrt{1-(\sin\theta + m\lambda f)^2}}. \tag{3}$$

In (2) and (3) f is assumed to be in units of lines/nm for simplicity. The angular dispersion $d\theta_m/d\lambda$ has units of radians/nm, and may be multiplied by $180/\pi$ to obtain units of degrees/nm.

Figure 1 illustrates angular dispersion from a ray point of view. Here light at two similar but distinct wavelengths λ and $\lambda + d\lambda$ is diffracted into the -1^{st} order at the two angles θ_{-1} and $\theta_{-1} + d\theta_{-1}$, respectively.

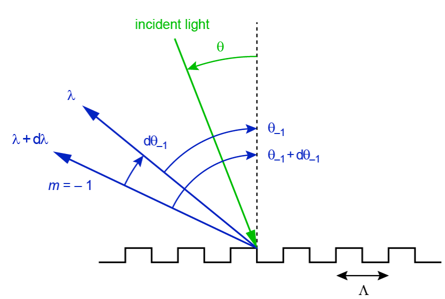

Figure 1

Calculating angular dispersion – an example:

The simple differential relations in (2) and (3) can be used to quickly and reasonably accurately estimate the angular variation $d\theta_{-1}$ that results from a small wavelength change $d\lambda$. For example, suppose light from a laser centered at 800 nm is incident on a 1480 lines/mm grating at an angle of 54º. The angle of diffraction for the –1st order at 800 nm is –22.02º, and at 810 nm it is –22.94º, so the angle change for $d\lambda$ = 10 nm is exactly $d\theta_{-1}$ = –0.92º. Approximating the angle change using (2), we calculate $d\theta_{-1}/d\lambda$ = –0.092 deg/nm, which also suggests a –0.92º angular change for a 10 nm wavelength change! Doing the same calculation for $d\lambda$ = 50 nm, we get an exact angular change of –4.66º and an approximate change from (2) of –4.60º, again demonstrating that the approximation is excellent, though not perfect, for some very practical numbers.

As with the Grating Equation, it is helpful to visualize angular dispersion by plotting it as a function of the angle of incidence for a certain wavelength. Figure 2 shows how the angular dispersion of the –1st order depends on angle of incidence at 800 nm for three different grating frequency values.

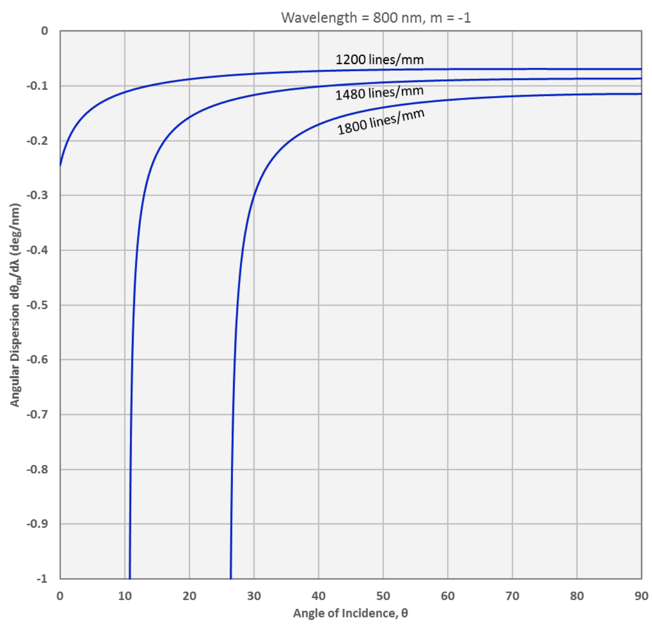

Figure 2

Note that the absolute value of the angular dispersion is larger for higher grating frequencies. Furthermore it increases very rapidly at smaller angles of incidence, as the angle of diffraction approaches –90º.

References:

[1] See PGL Technical Note "The grating equation"